with c being the speed of light. In other words the effective length increases as b increases or as the bandwidth of the device decreases. Similarly, for a transformer matched circulator it follows from (7) that

$$L_{\text{eff}} = Yc/2\omega_0. \tag{9}$$

For the circulators tested, (8) yields an effective length $L_{\text{eff}} =$ 12.7 cm in the first case, while (9) yields an effective length $L_{\text{eff}} =$ 6.8 cm in the second. Surprisingly, the effective length of the transformer matched circulator is smaller although it includes two additional quarter-wavelength sections of transmission line. The equivalent free-space diameter of the garnet disk was 5.2 cm or much less than the effective length of 12.7 cm for the simple circulator. Clearly, circulators behave in this respect like electrically long transmission devices.

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Power Generation and Efficiency in GaAs Traveling-Wave Amplifiers

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***Abstract*—The effect of the dielectric loading in a bidimensional GaAs traveling-wave amplifier (TWA) is investigated, with respect to the EM power generated by the structure and the efficiency of the dc to RF conversion. The validity of some usual approximations and assumptions is studied and a parameter, i.e., the power gain × efficiency product, is proposed as a useful tool for comparing the possible performances of TWA's.**

In this short paper the authors study the traveling-wave amplifier (TWA) structure proposed in Fig. 1 and seek an expression of the EM power generated by the electronic beam, which starts from the definition of an equivalent negative conductivity of the medium, which takes into account the effect of the RF charge ρ.

Following such an approach and using the solutions of dispersion

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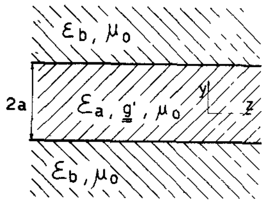

Fig. 1. Geometric behavior of the structure. The values of parameters used in the calculation of Re $[g_z']$ are: $\epsilon_a = 12$, $a = 1\ \mu$m, $n_0 = 10^{15}$ cm^{-3}, $v_0 = 1.5 \times 10^7$ cm/s, $\mu_y = 7200$ cm^2/(V·s), $\mu_z = -2400$ cm^2/(V·s).

relationship associated with the structure, the theoretical efficiency for different lateral loadings is obtained and the power gain × efficiency is indicated as a meaningful parameter of the TWA.

The propagation of EM waves in negative differential mobility (NDM) media has been studied by many authors [1]–[4] both in monodimensional and bidimensional approximation.

These papers deal prevalently with the modal solution of the structure which can sustain an infinite number of modes having different complex propagation constants, some of which correspond to growing waves. Following such an approach, the stability conditions were also ascertained which allow the use of the active structure as a TWA.

On the other hand, less attention has been paid to the energy as an approach which takes into account the power balance between the electronic stream and the EM field [5].

This is, moreover, a very important aspect of the matter because, starting from the premise that the real power "delivered" to the beam equals the real part of the Poynting vector flux entering a close surface delimiting a volume τ, the existence or nonexistence of amplifying waves is connected to the "sign" of the power "delivered" to the electronic beam. So, a formulation in terms of real power associated with the beam, except the kind of instability the beam can support, i.e., an absolute or convective instability, gives useful information on the properties of the structure when it is used as an amplifier.

Nevertheless, once it is found by the usual methods (Briggs [7], for instance) that instabilities are of the convective kind, the amount of generated real power for a fixed dc power dissipated by the device gives directly the efficiency of dc to RF power conversion.

The simple and well-known monodimensional model confirms that this point of view is essential for the evaluation of the amplifier characteristics and is a necessary complement of the solution of the problem in terms of phase and amplification constants associated with the space-charge wave.

In fact, an infinite medium with uniform characteristics is considered in the usual hypothesis of the small-signal traveling-wave analysis that follows.

1) Carrier mean free path is much shorter than wavelength.

2) Carrier lattice collision frequency is very large compared with the operating frequency.

3) Carrier drift velocity is parallel to the direction of wave propagation.

A compatible solution of the problem is [1]

$$\beta = \beta_e - j\beta_{cz}$$

where

$$\beta_e = \omega/v_0$$

$$\beta_{cz} = en_0\mu_z/\epsilon_0\epsilon_a v_0$$

with n_0 doping density, μ_z mobility, ϵ_a permittivity of the medium.

If μ_z is negative, as it can be in the case of GaAs devices biased above threshold, the corresponding wave grows as it travels at the "greatest growth rate" obtainable [2] in a monodimensional NDM structure, but no real RF power is generated since RF current density and electric field are 90° out of phase in the time domain, in accordance with the fact that no EM power flow leaves a volume τ [5] when ω is real.

Let us consider now the bidimensional structure shown in Fig. 1. Limiting values for power generation and efficiency have already been calculated for a slab like this, whose stability was verified with Briggs' criteria.

However, all these calculations start from the consideration that it is possible to assume, for the RF current density in the direction of wave propagation,

$$J_z = g_z E_z \tag{1}$$

where

$$g_z = e n_0 \mu_z. \tag{2}$$

Now we found that assumption (2) holds only in some cases, when the term ρv_0 is negligible in the linearized RF current expression

$$J = \rho v_0 + v \rho_0. \tag{3}$$

In order to obtain the effective expression of the power delivered to the EM field, we have removed the restriction (2) and calculated the expression of J_z, seeking a possible expression of the equivalent conductivity of the medium, to verify the validity range of (2).

In the reported 1)–3) hypothesis of small-signal traveling-wave analysis, and neglecting diffusion, the symmetric TM solutions (transverse-magnetic with respect to the z direction), of the form $\exp[j(\omega t - \beta z)]$ can be written, inside the GaAs slab,

$$E_{ya} = A \sin \alpha_a y \exp(-j\beta_z)$$

$$E_{za} = A \frac{k_c^2 - \beta^2}{j\alpha_a \beta} \cos \alpha_a y \exp(-j\beta_z)$$

$$H_{xa} = -A \frac{k_c^2}{\omega \mu_0 \beta} \sin \alpha_a y \exp(-j\beta_z) \tag{4}$$

where

$$k_c^2 = -j\omega\mu_0(j\omega\epsilon_0\epsilon_a + g_y)$$

with α_a, β related by the dispersion relationship of the structure [1]–[4].

Then in the linearized problem, the relationship between E_a and J_a is of the type

$$J_a = \bar{g}' \cdot E_a \tag{5}$$

where

$$\bar{g}' = \begin{pmatrix} g_y' & 0 \\ 0 & g_z' \end{pmatrix}$$

is a tensor whose components are field independent.

Now assuming

$$v = \bar{\mu} \cdot E_a \tag{6}$$

where

$$\bar{\mu} = \begin{pmatrix} \mu_y & 0 \\ 0 & \mu_z \end{pmatrix}$$

is the usual mobility tensor, we obtain from (3), using the continuity equation,

$$\rho_0 \left[\mu_y \frac{\partial}{\partial_y} E_{ya} + \mu_z \frac{\partial}{\partial_z} E_{za} \right] - j v_0 \rho \beta = -j\omega\rho. \tag{7}$$

Then (7) together with (4) and (3) gives

$$J_a = \rho_0 \mu_y E_{ya} y_0 + \rho_0 \mu_z E_{za} z_0 + \frac{\rho_0 \mu_y \alpha_a^2 + \rho_0 \mu_z (\beta^2 - k_c^2)}{\beta_e - \beta} \frac{\beta}{\beta^2 - k_c^2} E_{za} z_0. \tag{8}$$

So we have, for g_y' and g_z' defined in (5), the values

$$g_y' = \rho_0 \mu_y \tag{9a}$$

$$g_z' = \rho_0 \mu_z + \frac{\beta/\beta_e}{1 - \beta/\beta_e} \left[\frac{\rho_0 \mu_y \alpha_a^2}{\beta^2 - k_c^2} + \rho_0 \mu_z \right]. \tag{9b}$$

It is easy to see from (9) that, while g_y' has the usual expression $\rho_0 \mu_y$, g_z' differs from $\rho_0 \mu_z$ by a quantity which depends on frequency either directly, through β_e, or indirectly, through α_a and β, i.e., the solutions of the dispersion relationship for given values of the external permittivity ϵ_b.

Fig. 2 shows the equivalent conductivity Re $[g_z']$ versus frequency with external permittivity as a parameter, for the lowest order symmetric mode, obtained from (9b) after the computation of α_a and β from the dispersion relationship. The values of the other quantities are reported in the caption of Fig. 1. The broken lines in Fig. 2 correspond to values of frequency for which no zero-order physical solutions of the dispersion relationship are obtained.

We can observe that only if the external permittivity is high enough, the real part of g_z' equals the value corresponding to (2) and, in any case, that such an approximation falls down over a maximum frequency, which becomes lower while ϵ_b decreases.

In particular for $\epsilon_b = 1$, we obtain the smallest values for Re $[g_z']$, even if the gain of the corresponding wave is very close to β_{cz}, i.e., to that of the monodimensional case [4]; moreover, such values of Re $[g_z']$ indicate that the efficiency of the device will be very small.

A calculation was then made of the RF power generated in a volume (Fig. 3) for reasonable values of geometrical and electrical parameters.

Starting from

$$dP_{y,z} = \tfrac{1}{2} \mathrm{Re}[E_{y,z} \cdot J_{y,z}^*] d\tau \tag{10}$$

we can write

$$P_{\mathrm{tot}} = \tfrac{1}{2} g_y' \int_\tau |E_{ya}|^2 \, d\tau + \tfrac{1}{2} \mathrm{Re}[g_z'] \int_\tau |E_{za}|^2 \, d\tau. \tag{11}$$

Then using (4)

$$P_{\mathrm{tot}} = P_y + P_z = (S/8\beta_i) |A|^2 [1 - \exp(-2\beta_i L)] \cdot \{ g_y' (\Gamma_1 - \Gamma_2) + \mathrm{Re}[g_z'] |(\beta^2 - k_c^2)/\alpha_a \beta|^2 (\Gamma_1 + \Gamma_2) \} \tag{12}$$

where

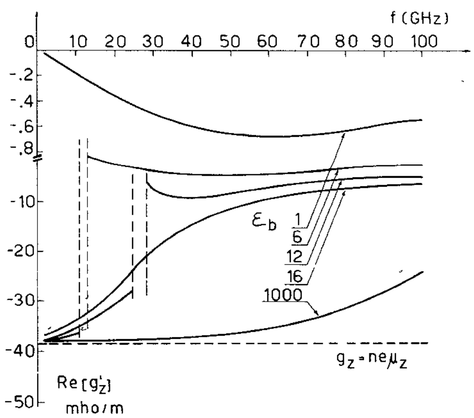

Fig. 2. Re $[g_z']$ versus frequency with external permittivity as a parameter.

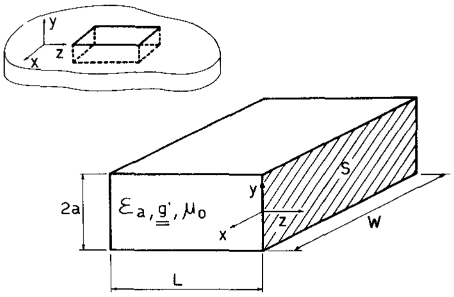

Fig. 3. Volume involved in the calculation of RF power generated. The values of parameters are: $a = 1\ \mu m$, $w = 1$ mm, $L = 10\ \mu m$, $E_0 = 6.0$ kV/cm, $E_p = 3.1$ kV/cm.

$$\Gamma_1 = \frac{\sinh 2\alpha_{aj}a}{2\alpha_{aj}a}$$

$$\Gamma_2 = \frac{\sin 2\alpha_{ar}a}{2\alpha_{ar}a}$$

$$\alpha_a = \alpha_{ar} + j\alpha_{aj}$$

$$\beta = \beta_r + j\beta_j.$$

The maximum amplitude of the electric field $|A|$ is determined by imposing that the field E_z cannot overcome the difference between the polarization field E_0 and the peak value E_p on the $v(E)$ characteristic, i.e., the threshold value for negative mobility: it was found that this assumption is more restrictive than the other one based on the maximum carrier density available [2].

Fig. 4 shows the behaviors of the different contributions to the total RF power (12) for $\epsilon_b = 16$: for comparison, the total power P_{t1} is also indicated in the hypothesis that Re $[g_z'] = \rho_0\mu_z$.

The algebraic sign reported near each curve means generated power if negative.

From Fig. 4 we can observe that the dissipated power P_y is negligible for an extended range of frequencies; its importance grows for higher frequencies, due to reduction of the generated power P_z.

As further observation, in this case the total power P_t is always less than the power P_{t1} calculated with Re $[g_z'] = \rho_0\mu_z$ and the two calculations give almost the same result in the lowest range of frequencies (see Fig. 2).

Fig. 5 shows the total power generated by the device for different lateral dielectric loadings: the right-hand scale gives the maximum theoretical efficiency available for a device length of 10 μm

$$\eta = P_{RF}/P_{dc} \tag{13}$$

where

$$P_{dc} = E_0\rho_0 v_0 LS$$

is the dc power dissipated along the device.

It appears that the efficiency η strongly depends on ϵ_b as the unitary gain β_j does [4] when the frequency is not too high, and then it is often unnecessary to use heavy dielectric loading, since, for certain frequencies, high efficiency in connection with high gain can be achieved by using lower values of ϵ_b (see Fig. 6). Moreover the choice $\epsilon_b = 1$ gives the smallest efficiency, as expected, and it can be observed, for instance, that for $\epsilon_b = 16$ the RF power behavior versus frequency is of the type constant $\times$ (frequency)$^{-1}$ [8] above $f \simeq$ 30 GHz.

Finally, Fig. 6 shows the parameter $Q = \eta \cdot 2\beta_j L$ versus frequency. Since it is desirable to have an amplifier both with high gain and high efficiency, the parameter Q can be considered as a sort of "figure of merit" of the device.

As can be easily derived from (12) and (13) and from the behavior of β_j versus frequency [2] for a given ϵ_b, Q does not depend on the

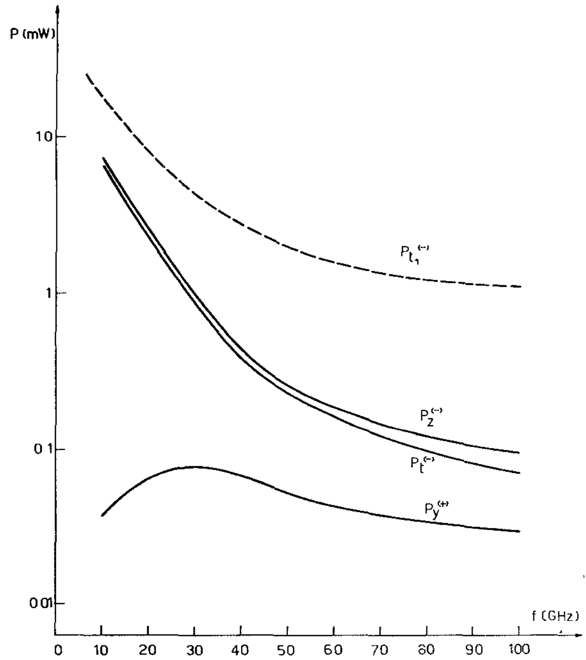

Fig. 4. Total power P_t, limit value P_{t1}, dissipated power P_y, generated power P_z versus frequency for $\epsilon_b = 16$.

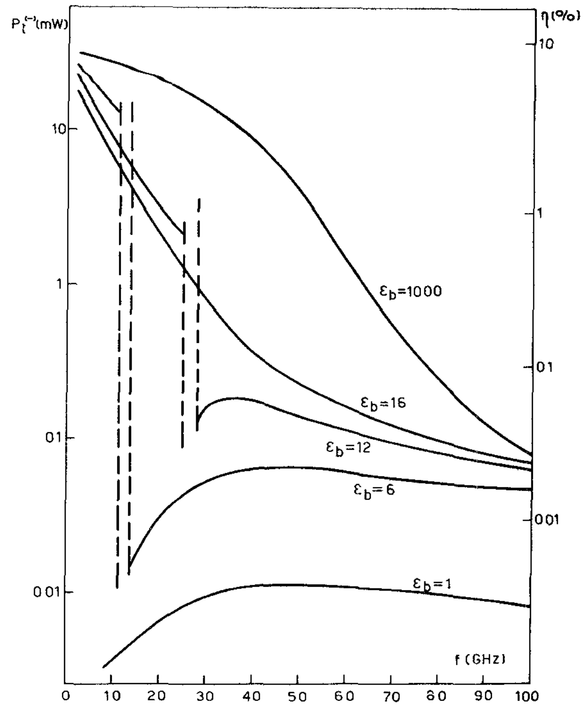

Fig. 5. Total power and efficiency versus frequency with ϵ_b as a parameter.

length of the device above a certain frequency, which increases with ϵ_b.

From Fig. 6, it is also clear that the best compromise between efficiency and gain, for a device 10 μm long, is achieved with a choice of ϵ_b in the range 12–16 and for frequencies below 40 GHz.

In conclusion a determination was made of the correct dependence between RF current and electric field in GaAs TWA's.

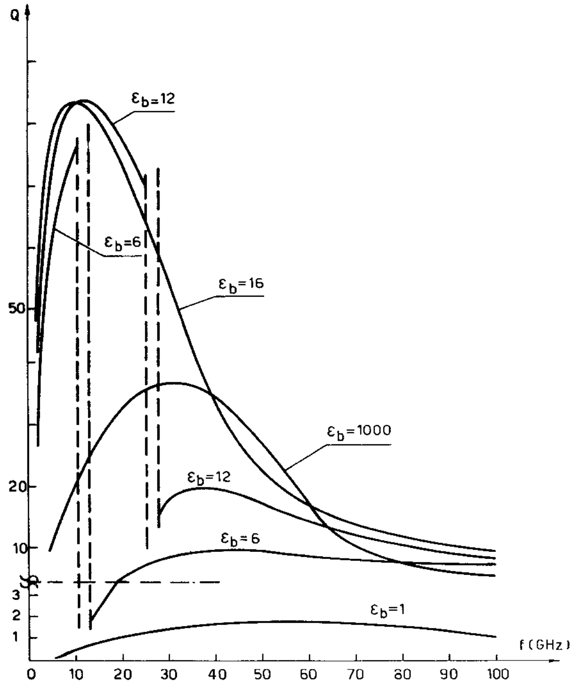

Fig. 6. Power gain × efficiency product (dB × percent) versus frequency with ϵ_b as a parameter.

This link depends on both frequency and external permittivity. It strongly influences power generation and efficiency of the device, which are normally less than the ones previously calculated.

Finally, a parameter was defined that is a valid tool in the choice of ϵ_b, in order to obtain the best compromise between gain and efficiency, in a certain frequency range.

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Letters

Dispersion of Nonlinear Elements as a Source of Electromagnetic Shock Structure

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Abstract—**Electromagnetic shock structure in nonlinear capacitance transmission lines can be resolved, and the energy losses associated with shock propagation explained, by including a resistance in series with the nonlinear capacitance. This resistance is inevitably present as the circuit representation of the nonvanishing relaxation time for the establishment of polarization in the nonlinear dielectric. Karbowiak and Freeman have dismissed this viewpoint as "not tenable!" This is a rebuttal of that statement.**

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The study of nonlinear electromagnetic wave propagation and of electromagnetic shock waves commenced with the pioneering work of Salinger [1] in 1923. Most modern authors, particularly in the vast quantum electronics literature, seem unaware of Salinger's work. The analysis of electromagnetic shock waves was revived and given its modern form around 1960 [2]. The field has since then given rise to a good many additional papers, some of which are cited in a recent analysis of the detailed structure of the electromagnetic shock [3].

Let us, for convenience, at this point specialize to the case of a nonlinear dielectric. In that case, different portions of a wavefront will see different values of the differential capacitance and will move with correspondingly different velocities. Thus wavefronts (or tails, depending on the sign of the nonlinearity) can sharpen, and shock formation results. Once a shock forms, the equations of motion of the shock, derived in the same way as in gas dynamics, do not correspond to energy conservation. This point is made in detail in a recent note by Karbowiak and Freeman [4]. The fact has, however, been widely understood in the field and is explicitly stated in [2].